Algebra and Functions

Quadratics

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $y = ax^2 + bx + c$:

y-intercept $= (0, c)$

Sum of roots $= \frac{c}{a}$

Product of roots $= -\frac{b}{a}$

When rewritten as $y = a(x - p)(x - q) \ \rightarrow p, q$ are roots/x-intercepts of the function.

When rewritten as $y = a(x - h)^2 + k \ \rightarrow \ (h, k)$ is the vertex of the function.

$x = h = -\frac{b}{2a}$ is the axis of symmetry for the quadratic function.

If $b^2 - 4ac > 0$, the quadratic has two real roots.

If $b^2 - 4ac = 0$, the quadratic has one real root (often referred to as a double root).

If $b^2 - 4ac < 0$, the quadratic has no real roots (or it has two complex roots).

Polynomials

Let $p(x)$ be a polynomial.

Factor theorem: $x - a$ is a factor $\leftrightarrow p(a) = 0$

Remainder theorem: when $p(x)$ is divided by $x - b$, the remainder of the division is equal to $p(b)$

If $(x - c)$ and $(x - d)$ are both factors of $p(x)$, then $(x - c)(x - d)$ is also a factor of $p(x)$.

$x - k$ is a factor $\leftrightarrow x = k$ is a root/x-intercept of the polynomial.

If complex number $x = a + bi$ is a root of polynomial $p(x)$, then $x = a - bi$ (the complex conjugate) must also be a root of $p(x)$.
